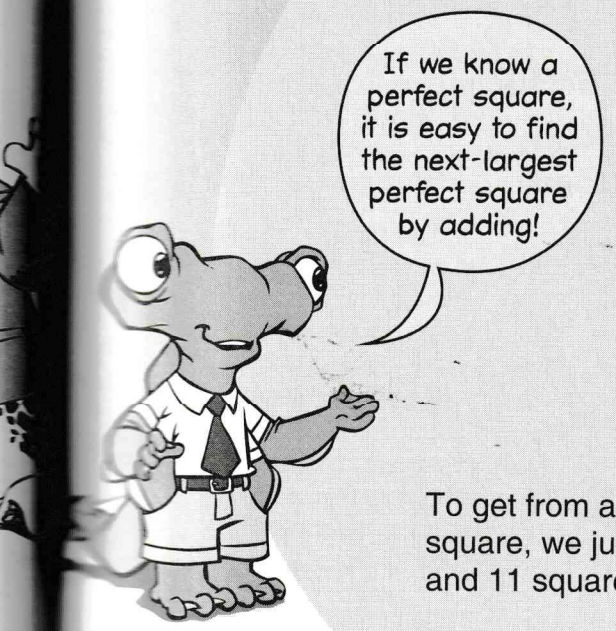
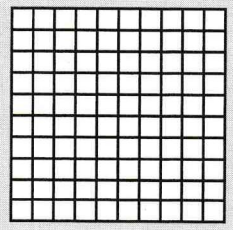


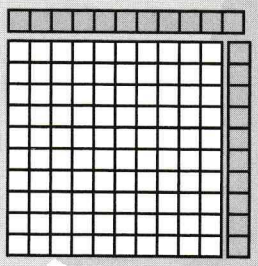
EXAMPLE | What is 11×11?

We already know $10\times10=100$.



To get from a 10×10 square to an 11×11 square, we just add 10 squares on the side and 11 squares on the top:

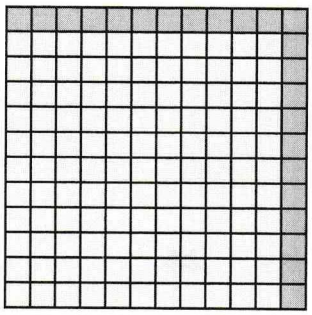
$11\times11=(10\times10)+(10+11)=100+21=\mathbf{121}$.



PRACTICE

34. Now that we know $11\times11=121$, what is 12×12?

34. _____



Use your answer above to find the squares below:

35. 13×13

35. _____

36. 14×14

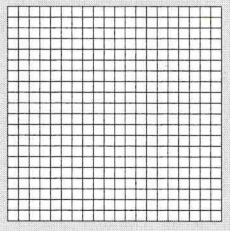
36. _____

37. How much larger is 15 squared than 14 squared?

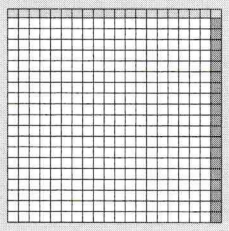
37. _____

EXAMPLE | What is 19×19?

We know that $20\times20=400$.



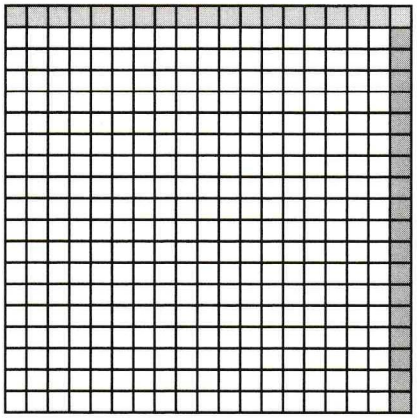
To get from a 20×20 square to a 19×19 square, we remove a row on top and a column on the side. To find 19×19, we subtract 20 and 19 from 400.



Subtracting 20 and 19 is the same as subtracting 39:
$19\times19=(20\times20)-20-19=400-39=\mathbf{361}$.

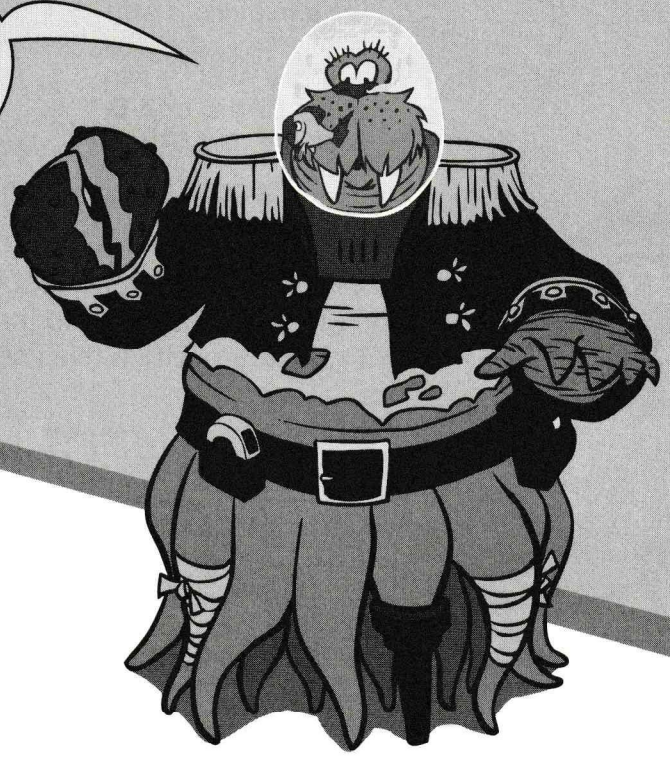
PRACTICE

38. Now that we know $19\times19=361$, what is 18×18? **38.** _____



39. What is $(18\times18)-(17\times17)$? **39.** _____

40. What is $(93\times93)-(92\times92)$? **40.** _____



PRACTICE | Complete these sequences of perfect squares.

41. $200\times200 =$ _____

$201\times201 =$ _____

$202\times202 =$ _____

42. $35\times35 =$ _____

$36\times36 =$ _____

$37\times37 =$ _____

43. $50\times50 =$ _____

$49\times49 =$ _____

$48\times48 =$ _____

44. $25\times25 =$ _____

$24\times24 =$ _____

$23\times23 =$ _____

45. ★ $29\times29 =$ _____

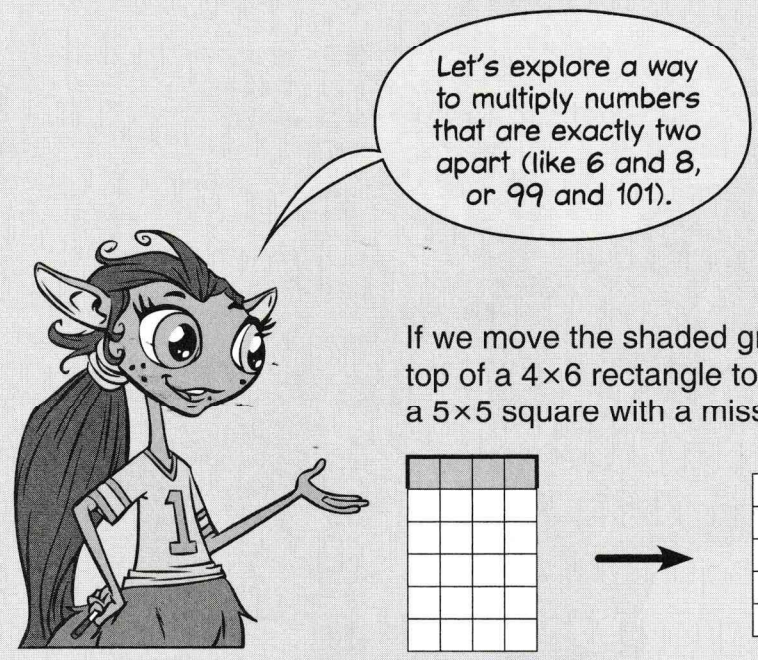
$30\times30 =$ _____

$31\times31 =$ _____

46. ★ $39\times39 =$ _____

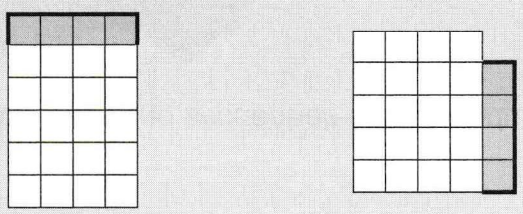
$40\times40 =$ _____

$41\times41 =$ _____



If we move the shaded group of squares from the top of a 4×6 rectangle to the side as shown, we get a 5×5 square with a missing corner:

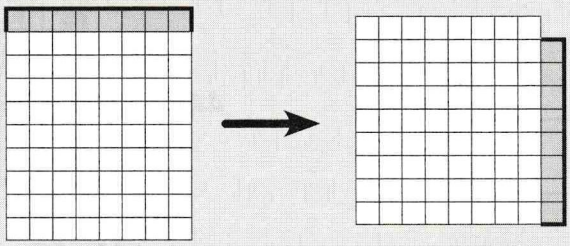
So, the area of a 4×6 rectangle is one less than the area of a 5×5 square:



$4\times6 \quad = \quad 5\times5-1$

In the same way, we can change any rectangle with side lengths that are two apart into a square that is missing one corner. The side length of the square is always the number between the side lengths of the rectangle.

For example, the area of an 8×10 rectangle is one less than the area of a 9×9 square.

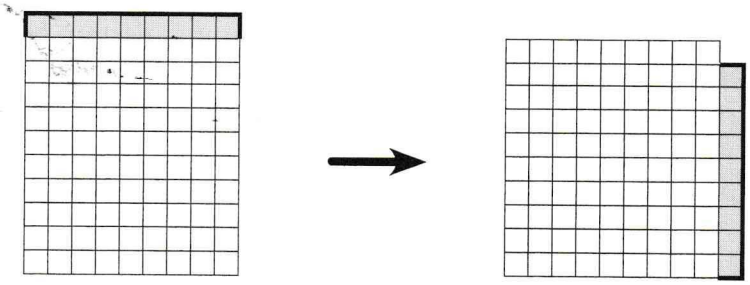


$8\times10 \quad = \quad 9\times9-1$

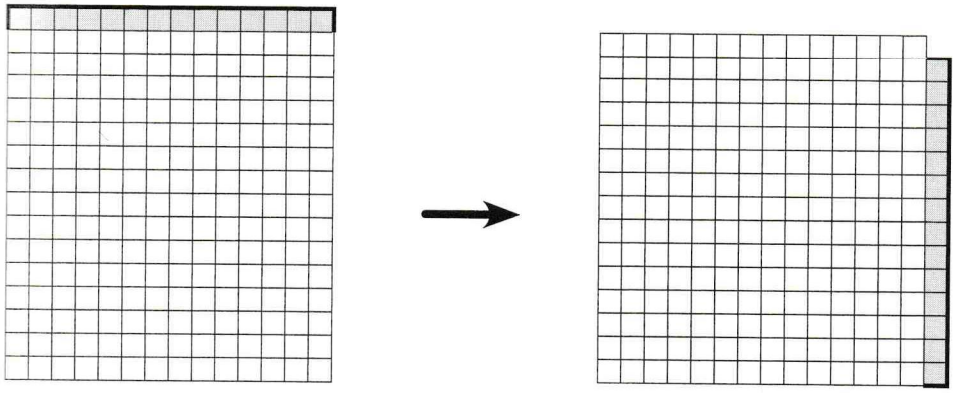
To multiply two numbers that are exactly two apart, we can square the number between them and subtract 1.

PRACTICE | Find the following products.

47. 9×11 **47.** _____



48. 14×16 **48.** _____



49. $19\times21 =$ _____ **50.** $81\times79 =$ _____

51. $24\times26 =$ _____ **52.** $49\times51 =$ _____

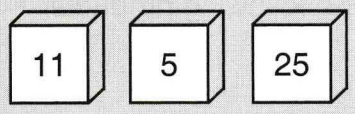
53. $64\times66 =$ _____ **54.** $46\times44 =$ _____

55. $801\times799 =$ _____ **56.** $3{,}001\times2{,}999 =$ _____

57. $8{,}999\times9{,}001 =$ _____ **58.** $104\times106 =$ _____

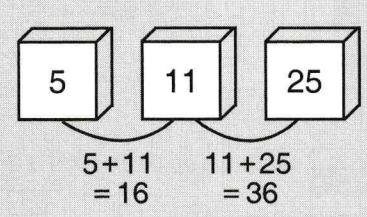
EXAMPLE

Rearrange the boxes below so that the sum of the numbers on any two neighboring boxes is a perfect square.



$5+25=30$ is not a perfect square, so the 5 and 25 boxes cannot be next to each other.
$11+5=16$ is a perfect square, so the 11 and 5 boxes can be next to each other.
$25+11=36$ is a perfect square, so the 25 and 11 boxes can be next to each other.

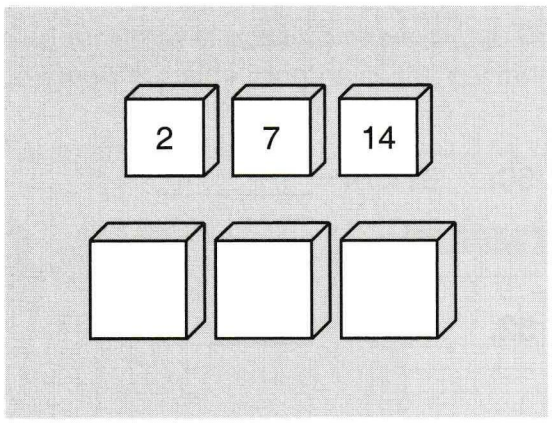
So, we can rearrange the boxes like this:



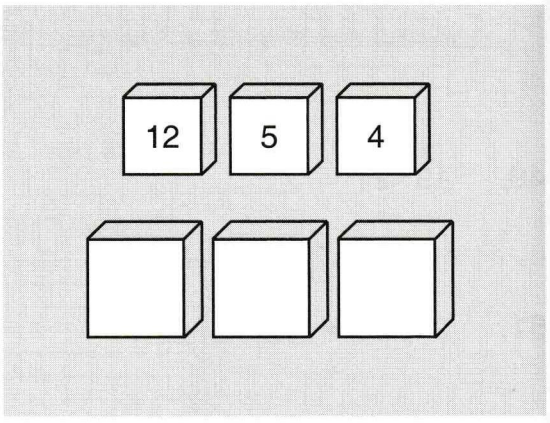
PRACTICE

Rearrange the boxes so that the sum of the numbers on any two neighboring boxes is a perfect square.

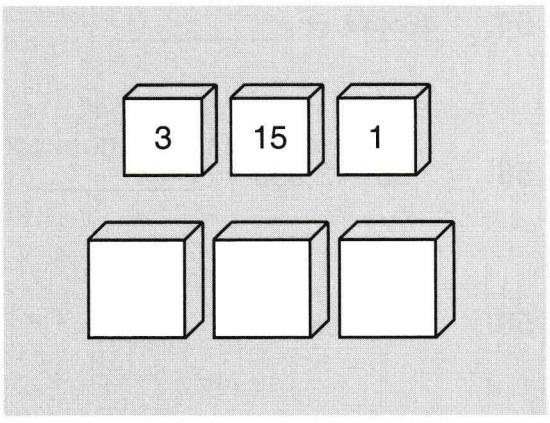
59.



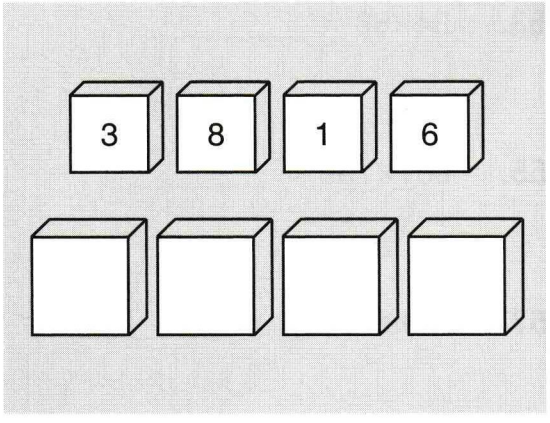
60.



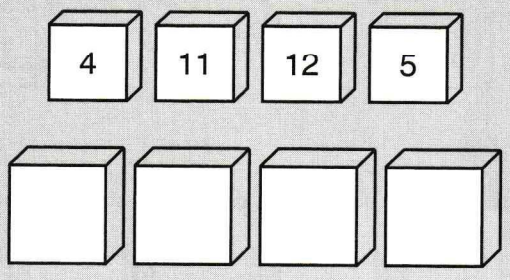
61.



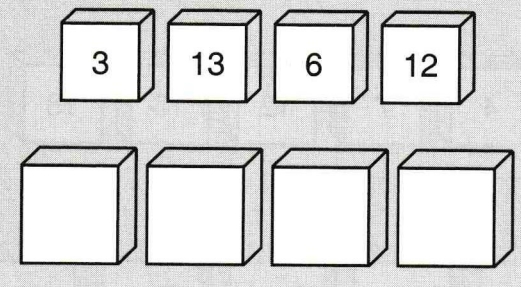
62.



63.



64.



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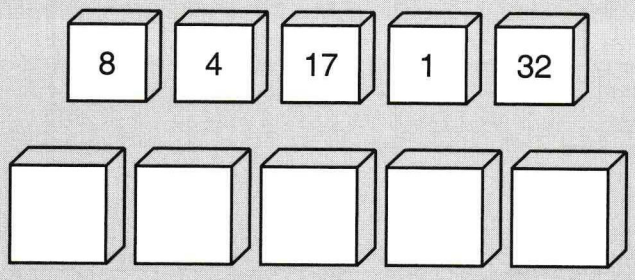
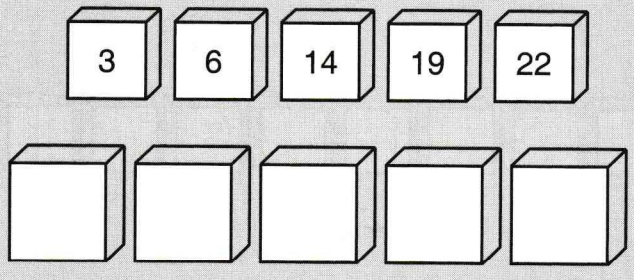


66.



67.
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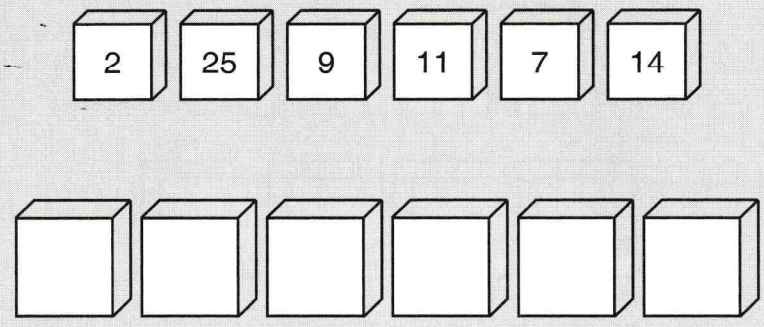

68.
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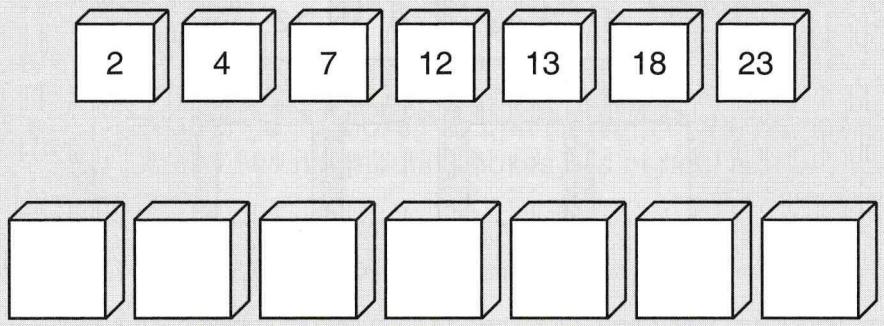

69.
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