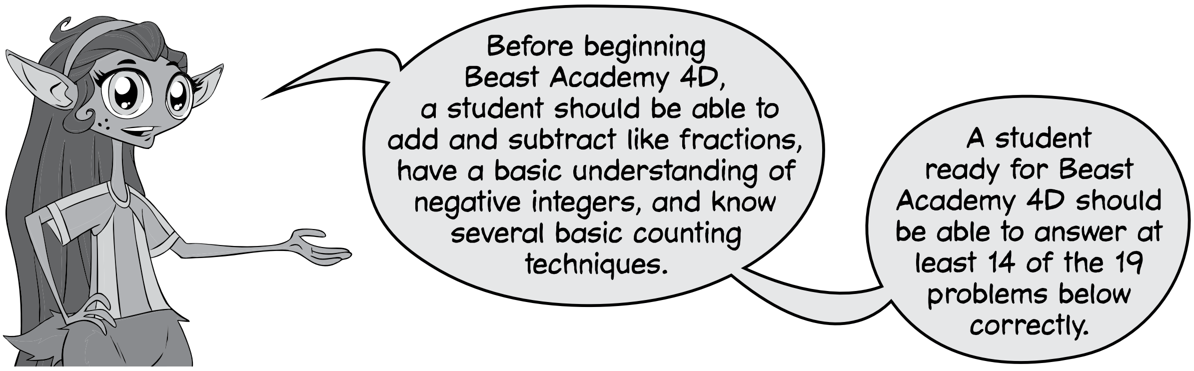
Are you ready for Beast Academy 4D?



Step 1. The student should try to answer every question without a calculator and without help.

Step 2. Check the student's answers using the solutions at the end of this document.

Step 3. The student should be given a second chance on problems that he or she answered incorrectly.

1. Fill in the blanks to create a four-digit number that is divisible by 2, 5, and 9.

8 _ 3 _

2. Fill in the blank to create a four-digit number that is divisible by 3 and 4.

2 4 7 _

3. List all the factors of 196.

3. ______________________________

4. Write the prime factorization of 1,350. Use exponents for repeated factors.

4. ______________________________

Compute each sum or difference below. Write your answer in simplest form, using whole or mixed numbers when possible.

5. $\frac{6}{7}-\frac{4}{7}=$

6. $7\frac{1}{4}-1\frac{3}{4}=$

7. $8\frac{2}{13}-3\frac{12}{13}=$

8. $5\frac{14}{17}+3\frac{6}{17}=$

9. How many centimeters are in the perimeter of an equilateral triangle whose sides are each $12\frac{7}{10}$ cm long?

9. __________

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Fill in the missing numerators and denominators to make equivalent fractions.

10. $\frac{1}{4} = \frac{25}{\_\_\_} = \frac{\_\_\_}{1{,}000}$

11. $\frac{14}{20} = \frac{\_\_\_}{10} = \frac{700}{\_\_\_}$

Complete each skip-counting pattern.

12. 15, 11, 7, _______, _______, _______, _______, _______, _______

13. -20, -17, -14, _______, _______, _______, _______, _______, _______

Fill in the missing entries in the grids below to make all the equations true.

14.

9	+	-2	=	
−	■	+	■	−
-3	−	-5	=	
=	■	=	■	=
	+		=	

15.

16	+		=	
−	■	−	■	−
	+	5	=	
=	■	=	■	=
	+	-6	=	-8

16. How many numbers are in the list below? **16.** _________

115, 118, 121, ... , 247, 250, 253

17. How many different arrangements of the letters in the word FIONA are possible, including F-I-O-N-A? **17.** _________

18. How many three-digit numbers have an odd hundreds digit, an even tens digit, and an odd ones digit? **18.** _________

19. Six monsters compete in a handball tournament. Each monster competes in exactly one game against each other monster. How many games are played? **19.** _________

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Solutions

1. Every multiple of 5 ends in 0 or 5. So, the ones digit of 8_3_ is either 0 or 5. However, we also know that 8_3_ is divisible by 2 (even), so 8_3_ cannot end in 5. The ones digit is 0.

Now, we have 8_30. For 8_30 to be divisible by 9, the sum of its digits must be a multiple of 9. The sum of the known digits is $8+3+0=11$. The only multiple of 9 that we can get by adding a digit (from 0 to 9) to 11 is $11+\boxed{7}=18$. Therefore, the hundreds digit is 7.

The four-digit number is **8730**.

2. For 247_ to be divisible by 3, the sum of its digits must be a multiple of 3. The sum of the known digits is $2+4+7=13$. The multiples of 3 that we can get by adding a digit (from 0 to 9) to 13 are $13+\boxed{2}=15$, $13+\boxed{5}=18$, and $13+\boxed{8}=21$. So, the ones digit is either 2, 5, or 8.

Every multiple of 4 ends in a two-digit multiple of four (00, 04, 08, 12, etc.) Since 247_ has a 7 in the tens place and is divisible by 4, we know that it must end in either 72 or 76. So, the ones digit is either 2 or 6.

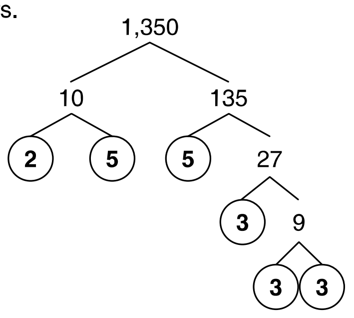
Only a ones digit of 2 meets both requirements, so the four-digit number is **2472**.

3. We begin by looking at the product pairs with the smallest factors and organize our work by writing the smaller number in each pair first.

196:
1×196
2×98
4×49
7×28
14×14

So, the factors of 196 are **1, 2, 4, 7, 14, 28, 49, 98, and 196.**

4. We use a factor tree to organize our work. We circle the prime factors.

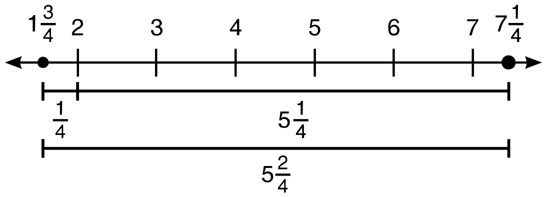


Writing these factors in order from least to greatest and using exponents for repeated factors, we have $1{,}350=\mathbf{2\times3^3\times5^2}$.

You may have used a different factor tree to arrive at the same final prime factorization.

5. Subtracting 4 sevenths from 6 sevenths leaves us with $6-4=2$ sevenths: $\frac{6}{7}-\frac{4}{7}=\mathbf{\frac{2}{7}}$.

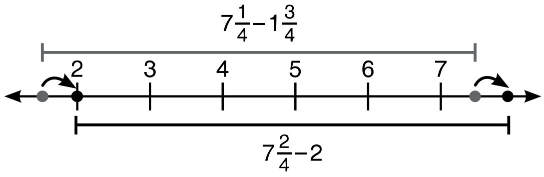
6. We count up. From $1\frac{3}{4}$ to 2 is $\frac{1}{4}$. Then, from 2 to $7\frac{1}{4}$ is $5\frac{1}{4}$ more.



So, the difference equals $\frac{1}{4}+5\frac{1}{4}=5\frac{2}{4}=\mathbf{5\frac{1}{2}}$.

— or —

$1\frac{3}{4}$ is $\frac{1}{4}$ less than 2. We add $\frac{1}{4}$ to both $7\frac{1}{4}$ and $1\frac{3}{4}$ to make the subtraction easier.



So, $7\frac{1}{4}-1\frac{3}{4}$ is equal to $7\frac{2}{4}-2=5\frac{2}{4}=\mathbf{5\frac{1}{2}}$.

— or —

To subtract $1\frac{3}{4}$, we can take away 2 and give back $\frac{1}{4}$.

$$\begin{aligned}7\tfrac{1}{4}-1\tfrac{3}{4}&=7\tfrac{1}{4}-2+\tfrac{1}{4}\\&=5\tfrac{1}{4}+\tfrac{1}{4}\\&=5\tfrac{2}{4}\\&=\mathbf{5\tfrac{1}{2}}.\end{aligned}$$

7. We cannot subtract 12 thirteenths from 2 thirteenths, so we regroup: $8\frac{2}{13}-3\frac{12}{13}=7\frac{15}{13}-3\frac{12}{13}=\mathbf{4\frac{3}{13}}$.

We could have instead used one of the strategies described in the previous problem to arrive at the same difference.

8. $5\frac{14}{17}+3\frac{6}{17}=8\frac{20}{17}=\mathbf{9\frac{3}{17}}$.

9. The perimeter of an equilateral triangle with side length $12\frac{7}{10}$ cm is

$$12\tfrac{7}{10}+12\tfrac{7}{10}+12\tfrac{7}{10}=36+\tfrac{21}{10}=36+2\tfrac{1}{10}=\mathbf{38\tfrac{1}{10}}\text{ cm.}$$

10. We can multiply the numerator and denominator of a fraction by the same number to make an equivalent fraction.

$$\frac{1}{4}\underset{\times25}{\overset{\times25}{=}}\frac{25}{\mathbf{100}}\underset{\times10}{\overset{\times10}{=}}\frac{\mathbf{250}}{1{,}000}$$

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11. We can multiply or divide the numerator and denominator of a fraction by the same number to make an equivalent fraction.

$$\frac{14}{20} \underset{\div 2}{\overset{\div 2}{=}} \frac{\mathbf{7}}{10} \underset{\times 100}{\overset{\times 100}{=}} \frac{700}{\mathbf{1{,}000}}$$

12. Each term in the pattern is 4 less than the one before it.

$$15,\ \overset{-4}{\curvearrowright}\ 11,\ \overset{-4}{\curvearrowright}\ 7,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_$$

We continue the pattern by subtracting 4's.

$$15,\ \overset{-4}{\curvearrowright}\ 11,\ \overset{-4}{\curvearrowright}\ 7,\ \overset{-4}{\curvearrowright}\ \mathbf{3},\ \overset{-4}{\curvearrowright}\ \mathbf{-1},\ \overset{-4}{\curvearrowright}\ \mathbf{-5},\ \overset{-4}{\curvearrowright}\ \mathbf{-9},\ \overset{-4}{\curvearrowright}\ \mathbf{-13},\ \overset{-4}{\curvearrowright}\ \mathbf{-17}$$

13. Each term in the pattern is 3 more than the one before it.

$$-20,\ \overset{+3}{\curvearrowright}\ -17,\ \overset{+3}{\curvearrowright}\ -14,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_,\ \_\_$$

We continue the pattern by adding 3's.

$$-20,\ \overset{+3}{\curvearrowright}\ -17,\ \overset{+3}{\curvearrowright}\ -14,\ \overset{+3}{\curvearrowright}\ \mathbf{-11},\ \overset{+3}{\curvearrowright}\ \mathbf{-8},\ \overset{+3}{\curvearrowright}\ \mathbf{-5},\ \overset{+3}{\curvearrowright}\ \mathbf{-2},\ \overset{+3}{\curvearrowright}\ \mathbf{1},\ \overset{+3}{\curvearrowright}\ \mathbf{4}$$

14. Step 1:

9	+	-2	=	**7**
−	■	+	■	−
-3	−	-5	=	**2**
=	■	=	■	=
12	+	**-7**	=	

Final:

9	+	-2	=	7
−	■	+	■	−
-3	−	-5	=	2
=	■	=	■	=
12	+	-7	=	**5**

15. Step 1:

16	+	**-1**	=	
−	■	−	■	−
	+	5	=	
=	■	=	■	=
-2	+	-6	=	-8

Step 2:

16	+	-1	=	**15**
−	■	−	■	−
18	+	5	=	
=	■	=	■	=
-2	+	-6	=	-8

Final:

16	+	-1	=	15
−	■	−	■	−
18	+	5	=	**23**
=	■	=	■	=
-2	+	-6	=	-8

16. This list counts by 3's, but the numbers are not all multiples of 3. To make the list easier to count, we subtract 112 from each number. This gives us a list of all multiples of 3 from 3 to 141:

$$\begin{array}{rrrcrrr} 115, & 118, & 121, & \ldots, & 247, & 250, & 253 \\ -112 & -112 & -112 & \ldots & -112 & -112 & -112 \\ \hline 3, & 6, & 9, & \ldots, & 135, & 138, & 141 \end{array}$$

Dividing each number in this new list by 3 gives us a list from 1 to 47.

$$\begin{array}{rrrcrrr} 3, & 6, & 9, & \ldots, & 135, & 138, & 141 \\ \div 3 & \div 3 & \div 3 & \ldots & \div 3 & \div 3 & \div 3 \\ \hline 1, & 2, & 3, & \ldots, & 45, & 46, & 47 \end{array}$$

This list is the same size as the original, and the numbers are counted for us!

So, there are **47** numbers in the original list.

17. We begin by choosing one letter to be the first. There are 5 choices for the first letter (F, I, O, N, or A).

Then, no matter which letter we chose first, there are always 4 choices for the second letter.

Similarly, no matter which two letters we chose first and second, there are always 3 choices remaining for the third letter.

Next, no matter which three letters we chose first, second, and third, there are always 2 choices remaining for the fourth letter.

Finally, once we have chosen the first four letters, there is only 1 remaining letter to be fifth.

All together, there are 5×4×3×2×1 = **120** ways to arrange the five letters in some order. You may have also written this as **5!** or "**5 factorial.**"

— or —

We consider placing the letters in these blanks: _ _ _ _ _ .

There are 5 possible spots we could choose for the F. No matter where F is placed, there are 4 remaining spots we could choose for I. No matter where F and I are placed, there are 3 remaining spots we could choose for O. No matter where F, I, and O are placed, there are 2 remaining spots we could choose for N. Finally, after placing the other four letters, there is just 1 spot remaining for the A.

All together, there are 5×4×3×2×1 = **120** ways to arrange the four letters in some order. You may have also written this as **5!** or "**5 factorial.**"

18. There are five odd digits: 1, 3, 5, 7, and 9.
There are five even digits: 0, 2, 4, 6, and 8.

So, we have 5 choices for the odd hundreds digit, 5 choices for the even tens digit, and 5 choices for the odd ones digit.

All together, we can create 5×5×5 = **125** such numbers.

19. Each game is played by one pair of monsters. When choosing a pair of monsters, we have 6 choices for the "first" monster and 5 choices for the "second" monster.

However, since "Grogg vs. Lizzie" is the same game as "Lizzie vs. Grogg," the order in which we pick the monsters does not matter. So, when we multiply 6×5 to get 30, we have counted each game *twice*.

Therefore, there are 30÷2 = **15** games in the tournament.